Supplementary Problems Set 1

1. Let us calculate the magnitudes and directions of the three charges at point P using the standard electric field formula.

   (a) To get that eletric field, we need to start by obtaining the distances.

   $$r_{1P} = \sqrt{8^2 + 4^2} = 8.9m \tag{1}$$

   Using the same reasoning, we can find r_{2P} and r_{3p} to be respectively 8.5m and 10m.

   Let us now calculate the magnitudes of the electric fields:

   $$E_1 = \frac{kq_1}{r_{1P}^2} = \frac{9 \cdot 10^9 \cdot 10^{-6}}{8.9^2} = 113.6N/C \tag{2}$$

   Using the same reasoning, we can find E_2 and E_3 to be respectively 249.1 N/C and 270 N/C. I have not kept the signs as I will be careful in my angles.

   Let us now find the angles at which all of the field lines are pointing. To do that, we can take the arctan of all the triangles to respectively get: 26.6°, 0° and 20.6°. These are NOT the direction in which the field lines are pointing. To get those, we must figure out that the field lines will point away from positive and towards negative. Using the angles we just found, we can get that the angles in which the field lines are pointing are −26.6°, 180° and 200.6°.

   The final step would be to take the sum of all the x and y components for all three vectors. Taking the sum of those vectors with the appropriate angles yields the net electric field $\boxed{\vec{E} = 400.2\hat{-i} + 145.9\hat{-j}N/C}$ or $\boxed{426\angle 200° N/C}$

   (b) To find the Force a charge q would, have, we only need to use $\vec{F} = q\vec{E}$. You can use either version of the electric field found above. The answer is: $\boxed{\vec{F} = 1.7 \cdot 10^{-5}\angle 20° N}$ or $\boxed{\vec{F} = 1.6 \cdot 10^{-5}\hat{\imath} + 5.85 \cdot 10^{-6}\hat{\jmath}N}$

2. (a) Since the charge on the left is weaker, the electric field will be 0 to the left of the left charge. Knowing that, we can write the equation that we need to solve to figure out the location of the point of interest. Notice that the problem gives you no charges.

   $$\frac{kq}{x^2} = \frac{k3q}{(x + 0.5)^2} \tag{3}$$

   Solving this equation for x yields two solutions: -0.19 (reject because between the charges) and 0.68 (accept). The electric field is $\boxed{68cm}$ left of the weaker charge.

(b) The simplest area where the potential is 0 will be between the two charges as they are opposite. Solving this equation:

$$\frac{kq}{x} = \frac{k3q}{(0.5 - x)} \tag{4}$$

Solving this equation yields $\boxed{0.125m}$ as x. Therefore one spot where the potential is 0 is 0.125 to the right of the weaker charge. Another spot where the potential could be 0 is $\boxed{0.25m}$ to the left of the weaker charge.

(c) The electric field must be on the line given that all the components must cancel. Should you go a little higher or a little lower, it would be impossible for the 2 vectors to be antiparallel (ie cancel out). For potential they don't have to be. Given that is is a scalar, you just need the magnitudes to be opposite. That can happen in many places.

3. (a) Strategy: Given that we have the velocity in the x direction and the length of the plates, a simple division will give us the time spent in between the plates.

$$\frac{l}{v} = \frac{0.03}{2 \cdot 10^7} = 1.5 \cdot 10^{-9} \tag{5}$$

The electron spends $\boxed{1.5ns}$ in the region with the plates.

(b) Strategy: To get the vertical displacement, we will need to get the acceleration it will feel when in the plates. From that acceleration, we can use kinematics to obtain the vertical displacement in the plates.

$$F = ma = qE \rightarrow a = \frac{qE}{m} = 3.5 \cdot 10^{15} \tag{6}$$

Recall that since it is an electron, the force (and the acceleration) are directed downwards. Therefore $\boxed{\vec{a} = 3.5 \cdot 10^{15} - \hat{\jmath} m/s^2}$.

Next, we need to get the velocity. Using kinematics, we obtain that:

$$y = y_0 + v_{0y} \cdot t + \frac{a_y t^2}{2} = 0 + 0 + \frac{3.5 \cdot 10^{15} \cdot (1.5 \cdot 10^{-9})^2}{2} = 3.94mm \tag{7}$$

The answer is therefore that the deflection is $\boxed{3.94mm}$ downwards.

(c) Strategy: Since we know the acceleration and the time, this is a straightforward kinematics problem.

$$v_y = a_y \cdot t = 3.5 \cdot 10^{15} \cdot 1.5 \cdot 10^{-9} = 5.25 \cdot 10^6 \tag{8}$$

Since they want velocity, which is a vector, remember to take into account the x component. Therefore, the velocity is $\boxed{\vec{v} = 2 \cdot 10^7 \hat{\imath} + 5.25 \cdot 10^6 - \hat{\jmath} m/s}$

(d) Strategy: Since they want the vertical distance where the electron hits the screen, we must add the vertical displacement found in part B to the vertical displacement that it will have becomes it comes out of the plates at an angle below the horizontal. Simple kinematics will work once again, but we will need a new time: the time it takes to go from the plates to the screen. That time will be $t_2 = 0.14/(2 \cdot 10^7) = 7ns$

$$y = v_y \cdot t_2 = 5.25 \cdot 10^6 \cdot 7 \cdot 10^{-9} = 36.7mm \tag{9}$$

Now that we have the vertical distance traveled after the plates, we just need to add the vertical distance travelles in the plates to get the total vertical distance: $3.94 + 36.7 = \boxed{40.7mm}$

4. Strategy: This is a conservation of energy problem. It is therefore necessary to calculate all the initial potential energy and the final potential energy. The difference in the two will yield the kinetic energy and therefore the speed.

$$U_i = \frac{kqQ}{r} = \frac{k3.1 \cdot 10^{-6} \cdot 3.1 \cdot 10^{-6}}{9 \cdot 10^{-4}} = 96.1J \tag{10}$$

$$U_f = \frac{kqQ}{r} = \frac{k3.1 \cdot 10^{-6} \cdot 3.1 \cdot 10^{-6}}{2.5 \cdot 10^{-3}} = 34.6J \tag{11}$$

Now that we have both potential energies, we can figure out the increase in kinetic energy.

$$96.1 - 34.6 = \frac{mv^2}{2} = \frac{2 \cdot 10^{-5} v^2}{2} \to v = 2.48 \cdot 10^3 m/s \tag{12}$$

From conservation of energy, we obtain that the speed is $\boxed{2.48 \cdot 10^3 m/s}$

5. (a) Strategy: Just take the sum of the potential energies.

$$U_{12} + U_{13} + U_{23} = \frac{kq_1q_2}{r_{12}} + \frac{kq_1q_3}{r_{13}} + \frac{kq_2q_3}{r_{23}} = 6.4 \cdot 10^{-3} J \tag{13}$$

The intial potential energy is $\boxed{6.4 \cdot 10^{-3} J}$

(b) Strategy: To get the potential at P, we need to sum all the potentials at P... We can start by calculating the distances which are respectively: 8.9, 8.5 and 10 m.

$$V_P = V_{1P} + V_{2P} + V_{3P} = \frac{kq_1}{r_{1P}} + \frac{kq_2}{r_{2P}} + \frac{kq_3}{r_{3P}} = -3.8 \cdot 10^3 V \tag{14}$$

The potential at P is $\boxed{-3.8 \cdot 10^3 V}$

(c) Strategy: Since we have just found the potential at P in equation 14, we can get the potential energy at P by simply using $U = qV$.

$$U = qV_P = -4 \cdot 10^{-10} \cdot -3.8 \cdot 10^3 = 1.5 \cdot 10^{-6} J \tag{15}$$

The potential energy of that charge at P is $\boxed{1.5 \cdot 10^{-6} J}$

(d) Strategy: We will solve this problem in the exact same way as we solved part: by taking the sum of all three potentials.

$$V_0 = \frac{kq_1}{r_{10}} + \frac{kq_2}{r_{20}} + \frac{kq_3}{r_{30}} = -1.7 \cdot 10^4 V \tag{16}$$

The potential at the origin is $\boxed{-1.7 \cdot 10^4 V}$

(e) Strategy: Since we have just found the potential at point P in part b and we also have the potential at the origin from part d, we can use the difference in their potentials to figure out the work done or the change in potential energy.

$$W = q_4 \Delta V = -4 \cdot 10^{-10} \cdot \left(-1.7 \cdot 10^4 - -3.8 \cdot 10^{-6}\right) = 5.3 \cdot 10^{-6} J \tag{17}$$

The work done to bring q_4 from P to the origin will require positive work as the potential at the origin is less negative. The work done is $\boxed{5.3 \cdot 10^{-6} J}$

6. Strategy: Since we are looking for the work spent between two different charge configurations, we can take the difference in the potential energies when all 4 charges are present compared to when only 2 charges are present. The difference in those energies will be the cost of moving the 2 charges away. If we calculate the distance between the charges in the corner, we get 8.9 cm.

$$U_i = U_{12}+U_{13}+U_{14}+U_{23}+U_{24}+U_{34} = \frac{kq_1q_2}{0.04}+\frac{kq_1q_3}{0.089}+\frac{kq_1q_4}{0.08}+\frac{kq_2q_3}{0.08}+\frac{kq_2q_4}{0.089}+\frac{kq_3q_4}{0.04} = 12J \tag{18}$$

Compared to the final potential energy using the same charge assignment:

$$U_f = U_{13} = \frac{kq_1q_3}{0.089} = 1J \tag{19}$$

Therefore, the work work required to bring the two $4\mu C$ charges to ∞ is $1-12 = \boxed{-11J}$

7. (a) Strategy: We are looking for the potential energy between where the electron started and where it finished. Since energy is conserved, we can figure out that all of the kinetic energy lost, must have been gained by the potential energy.

$$\Delta K = \Delta U = \frac{mv_f^2}{2} - \frac{mv_i^2}{2} = 1.8 \cdot 10^{-20} - 1.14 \cdot 10^{-17} = -1.14 \cdot 10^{-17} J \tag{20}$$

To get the potential change between the two, we just need to do some algebra:

$$\Delta K = \Delta U = -1.14 \cdot 10^{-17} = q\Delta V \rightarrow \Delta V = 70.9V \tag{21}$$

The potential difference between then two is therefore $\boxed{70.9V}$.

(b) Since the electron loses kinetic energy (gains potential) as it goes towards the right, there must be a negative charge to the right. Using that logic, the potential must be lower on the right (due to the presence of a negative charge that is slowing down our initial electrons) and higher at the origin.

8. Strategy: The only uncertain part is where the dipole moment is pointed. Since the dipole moment points in the opposite direction as field lines, it is pointed at 37 degrees with respect to the field lines. Therefore the angle in the formula will be 37.

$$\tau = pEsin(\theta) = qdEsin(\theta) = 3 \cdot 10^{-7} \cdot 0.05 \cdot 4 \cdot 10^{3} \cdot sin(37) = 3.6 \cdot 10^{-5} Nm \tag{22}$$

Since torque is a vector, the net torque will be $\boxed{\vec{\tau} = 3.6 \cdot 10^{-5} -\hat{k} Nm}$ assuming a normal coordinate system ($+x$ towards the right, $+y$ towards the top)